Motion Blur and Photo Viewing

Photography is about recording a moment in time. This moment in time, the exposure, is not instantaneous but has a finite value, which ranges from thousandths of a second to minutes or, even, hours. The objects within a photograph, whether the subject or not, may or may not be stationary for the duration of the exposure. For non-stationary objects, the relative motion is critical to the resulting photograph; the shutter speed, velocity (speed and direction), camera-to-object distance, final image viewing ratio and the lens zoom are factors that affect the resulting photograph. Artistic choice guides whether the motion provides blurring or not. However, the photographic means of achieving the choice is technical.

For the purpose of this brief analysis, I have made a few assumptions and simplifications.

1. The photograph orientation is landscape. That is, the image is wide, relative to the height.
2. The object moves at a constant velocity, horizontally and in the plane of focus. The motion is at the center of the camera view.
3. The pixels used in taking the photograph are also used to produce the image (i.e. the image is not cropped).
4. The image viewing ratio (viewing distance relative to an image dimension) was important, not the size of the image.
5. Viewer resolution of the image is assumed to be 1/3000 of the image viewing distance.
6. The photographed displacement of the object (i.e. motion while the shutter is open) should be no more than the viewer resolution of the printed image.

Figure 1 and Table 1 show the variables and definitions that apply to photographing the object.

Figure 2 and Table 2 show the variables and definitions that apply to the viewing of the photograph image.

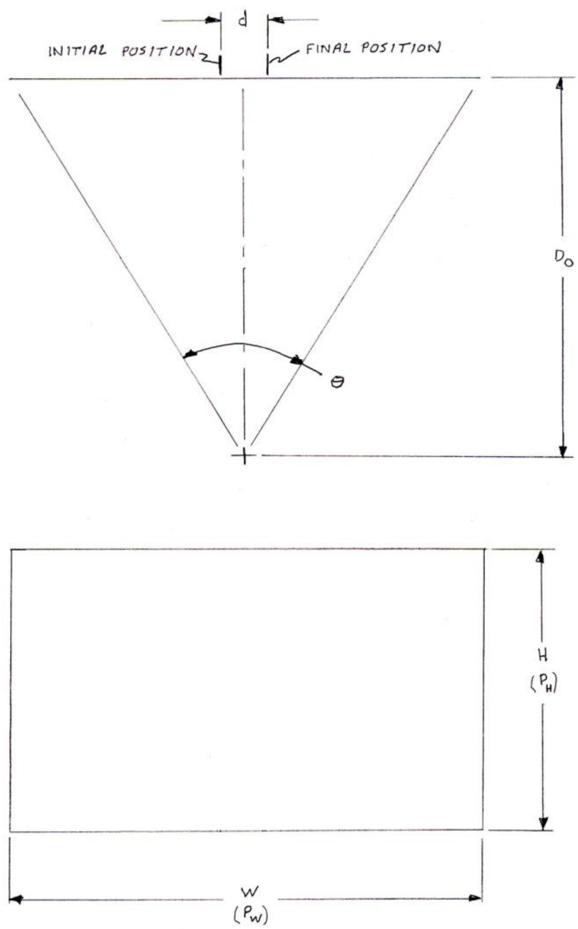

Figure 1
Camera/Object Variables

Variable	Description and Units
D_O	Object Distance (feet)
d	Object Displacement (feet)
d_P	Object Displacement (pixels)
t	Shutter Time (Time for Object Displacement) (seconds)
t_W	Object Traverse Time (seconds)
P_W	Camera Pixel Count, Width (pixels)
P_H	Camera Pixel Count, Height (pixels)
θ	Camera View Angle (degrees)
W	View Width at Focus/Object Distance (feet)
v	Object Velocity

Table 1
Camera/Object Variable Definitions

The View Width at Object Distance is calculated as follows.

Equation 1. $W = 2(D_O)(\tan(\theta/2))$

The Object Displacement, the Object Velocity and the time are defined in Equations 2a and 2b.

Equation 2a. $v = d/t$

Equation 2b. $d = (v)(t)$

The Object Displacement, in pixels, is proportional to the ratio of the Object Displacement and the View Width. See Equation 3.

Equation 3. $d_p = (d/W)P_W$

By substituting Equations 1 and 2b into Equation 3, the displacement, in pixels, is defined by,

Equation 4. $d_P = [(v)(t)(P_W)]/ [\ 2(D_O)(\tan(\theta/2))]$

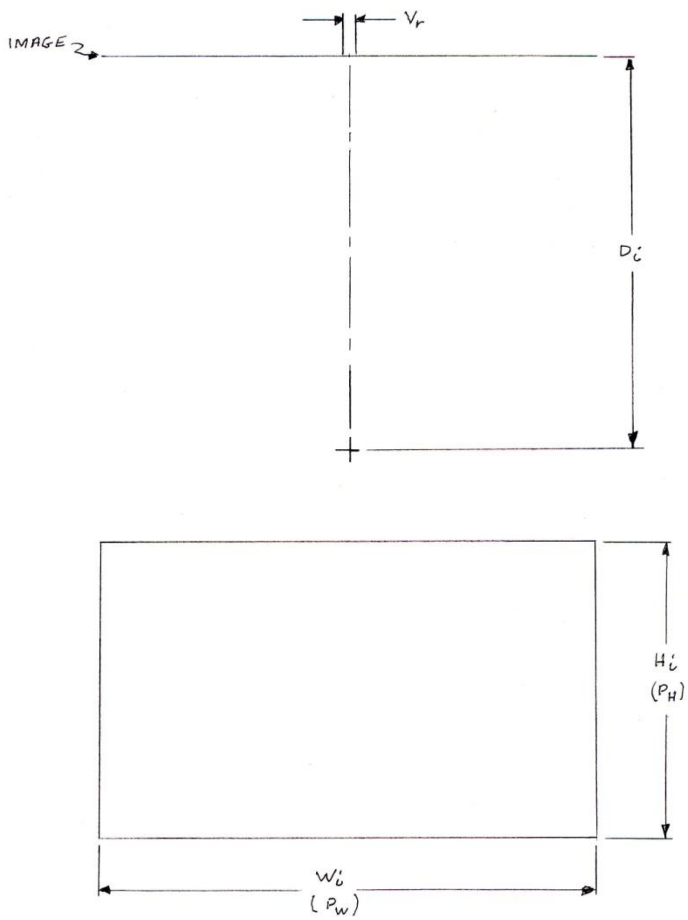

Figure 2
Image/Viewing Variables

Variable	Description and Units
D_i	Viewing Distance from Image (inches)
H_i	Vertical Image Size (inches)
W_i	Horizontal Image Size (inches)
R	Image Viewing Ratio
V_r	Visual Resolution (inches)
V_{rp}	Resolution (pixels)

Table 2
Image/Viewing Variable Definitions

Often, the Image Viewing Ratio is based upon the viewing distance and the diagonal length of the image. To simplify the calculations, I chose to define the Image Viewing Ratio as the ratio between the viewing distance and the horizontal length of the image (Equation 5); later, I'll show the correlation between the two ratios.

Equation 5. $R = D_i/W_i$

Allowing the Viewing Resolution factor of 3000, the Visual Resolution is calculated.

Equation 6. $V_r = D_i/3000$

Next, the Visual Resolution in pixels is calculated.

Equation 7. $V_{rP} = (D_i/W_i)(P_W/3000)$

Subtituting Equation 5 into Equation 7, yields the Visual Resolution (Equation 8), in pixels, based upon the Image Viewing Ratio and the Image Width (in pixels).

Equation 8. $V_{rP} = (R)(P_W)/3000$

For clarity, the photographed displacement of the object should be less than or equal to the viewer resolution. Thus, $d_P \le V_{rP}$.

Equation 9. $[(v)(t)(P_W)]/ [2(D_O)(\tan(\theta/2))] \le (R)(P_W)/3000$

Equation 10. $t \le [(R)(D_O)(\tan(\theta/2))]/[1500(v)]$

Thus, the shutter time is defined in terms of the Viewing Ratio, the Object Distance, the Camera View Angle and the Object Velocity. Typically, photographers use the inverse of the shutter time, so Equation 11 solves for "1/t".

Equation 11. $1/t \ge [1500(v)]/ [(R)(D_O)(\tan(\theta/2))]$

A sample plot of Equation 11 is presented. Table 3 correlates the lens length with the view angle. For the four curves of the plot, the following values were used.

1. Object Velocity (v) – 1 mph, 60 mph, 100 mph and 200 mph
2. Viewing Ratio (R) – 2.0
3. Object Distance (D_O) – 1,000 feet

Figure 3 shows the plots of shutter speed versus lens length.

Lens Length (mm)	View Angle (θ) (degrees)
28	65
35	54
50	40
70	29
100	20
105	19
135	15
150	14
200	10
250	8
300	7

Table 3
Lens Length and View Angle

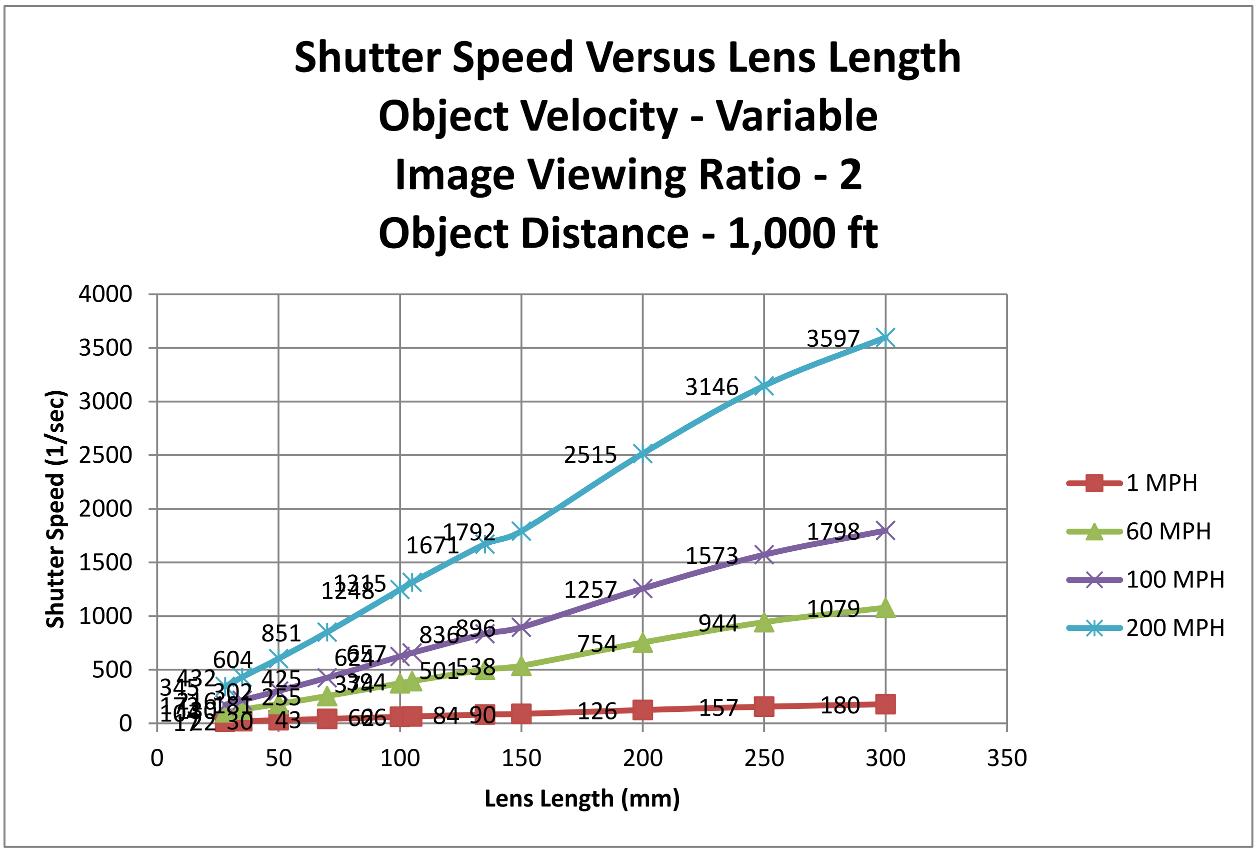

Figure 3
Shutter Speed Versus Lens Length for Various Object Velocities

Figure 3 shows the shutter speed when the object displacement equals the visual resolution of the image. Thus, the moving object would not be blurred more than the visual resolution in the image. From the curves of Figure 3, it can be seen that,

1. For an object speed of 200 mph and a 200mm lens, the shutter speed should be 1/2515 second.
2. If the object speed were 60 mph and a 300mm lens were used, then the shutter speed would be 1/1079.

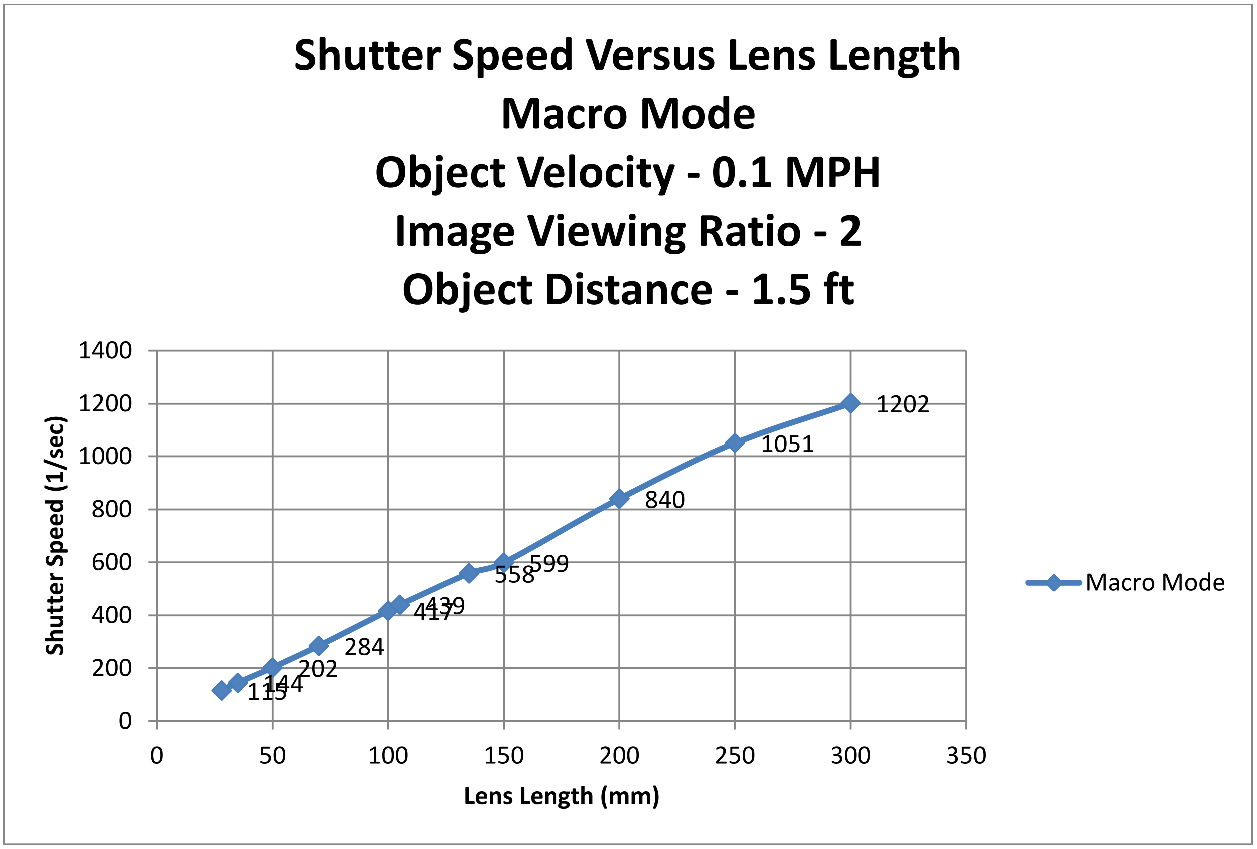

Figure 4
Shutter Speed Versus Lens Length for Close (Macro) Focus

Figure 4 shows the shutter speed for close focus photography. The velocity of 0.1 mph is 0.15 inches per second. Though a seemingly slow velocity, the shutter speed for "stopping the action" is significant, especially as a longer lens is used.

An interesting perspective of Equation 11 is found through the View Width, which is Equation 1. The term "$D_O(\tan(\theta/2))$" is found in Equation 1. By rearranging Equation 1, the term "$D_O(\tan(\theta/2))$" is defined by "W", which is the View Width at the Object Distance.

Equation 12. $(D_O)(\tan(\theta/2) = W/2$

Substituting Equation 12 into Equation 11 yields the following.

Equation 13. $1/t \geq [3000(v)]/ [(R)(W)]$

From Equation 13, it is seen that, if R and W are constant, then the shutter speed is dependent upon the object velocity, only. This is shown in Figure 5. For Figure 5, the following assumptions were used.

1. Object Velocity (v) – 60 mph
2. Object Distance (D_O) – 50 – 1,000 feet
3. Image Viewing Ratio (R) – 2.0
4. Two Lens configurations were considered
   a. 300mm lens
   b. 300mm zoom lens such that the viewing width is constant for each Object Distance. The 122.325 foot viewing width was calculated for a 300mm lens and 1,000 foot object distance.

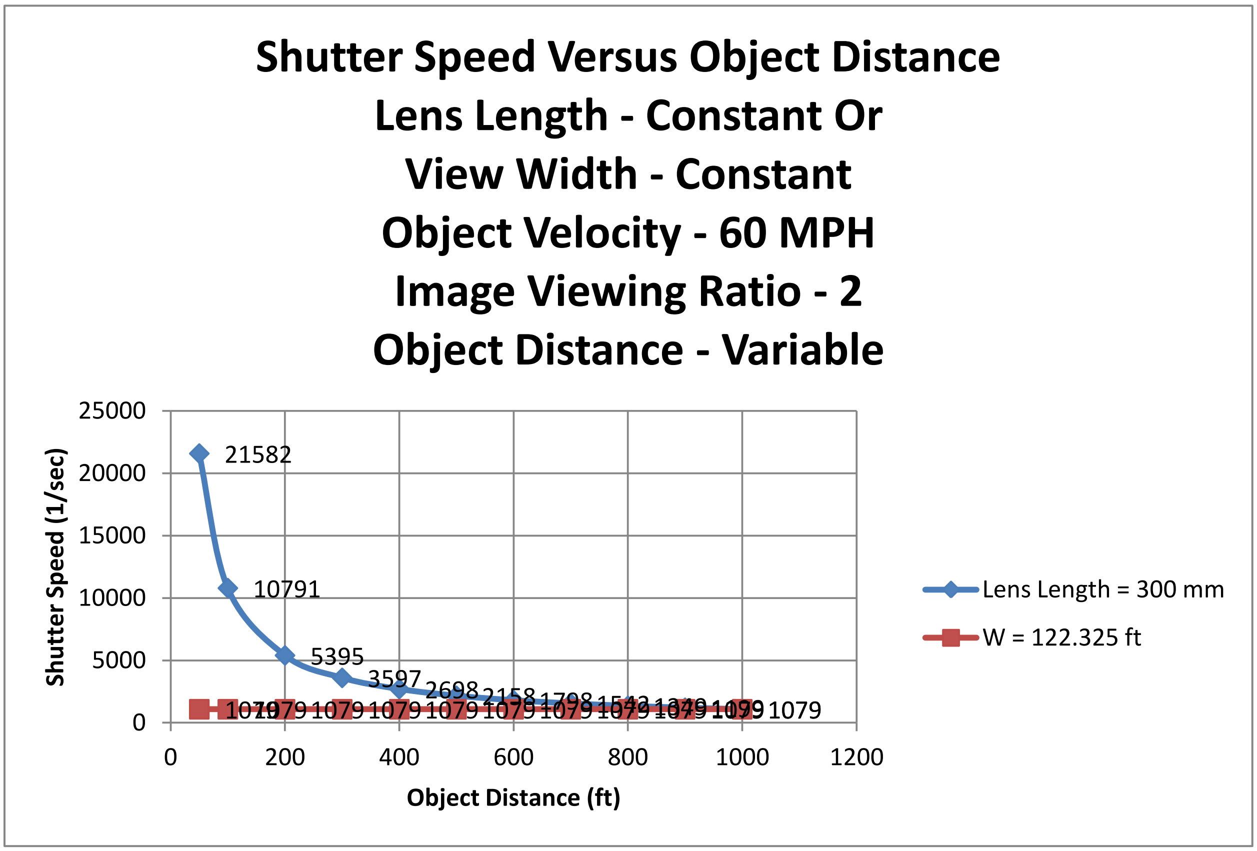

Figure 5
Shutter Speed Versus View Width

The preceding calculations are grand; they provide insight into the technical accomplishment of the artistic choice of blur. However, how often does one know the speed of a race car and its distance from the camera? How fast do seagulls, pelicans or buzzards flap their wings? When the shutter opens for a split second, how far away are the birds? The preceding equations are lacking practicality for field use.

Amazingly, another simple equation will solve these problems.

Equation 14. $t_W = W/v$

Dividing the View Width by the velocity of the object is time t_W. This is the time for the object to traverse the width of view that is being photographed. Next, substitute Equation 14 into Equation 13.

Equation 15. $1/t \geq 3000/ [(R)(t_W)]$

With Equation 15, the shutter time is dependent upon the Viewing Ratio and the Object Traverse Time t_W. The values of the lens length, the object distance and the View Width are eliminated.

The Object Traverse Time can be quickly estimated in the field for each photographic condition.

Table 4 is the data for the plot of Figure 6.

Figure 6 shows a plot of the shutter time versus the Object Traverse Time.

Object Traverse Time	Viewing Ratio	$1/t = 3000/ [(R)(t_W)]$
(seconds)		(1/seconds)
0.1	2	15,000
0.2		7,500
0.3		5,000
0.4		3,750
0.5		3,000
0.6		2,500
0.7		2,143
0.8		1,875
0.9		1,667
1		1,500
2		750
3		500
4		375
5		300
6		250
7		214
8		188
9		167
10		150
11		136
12		125
13		115
14		107
15		100

Table 4
Data for Figure 6
Object Traverse Time, Viewing Ratio and Shutter Time

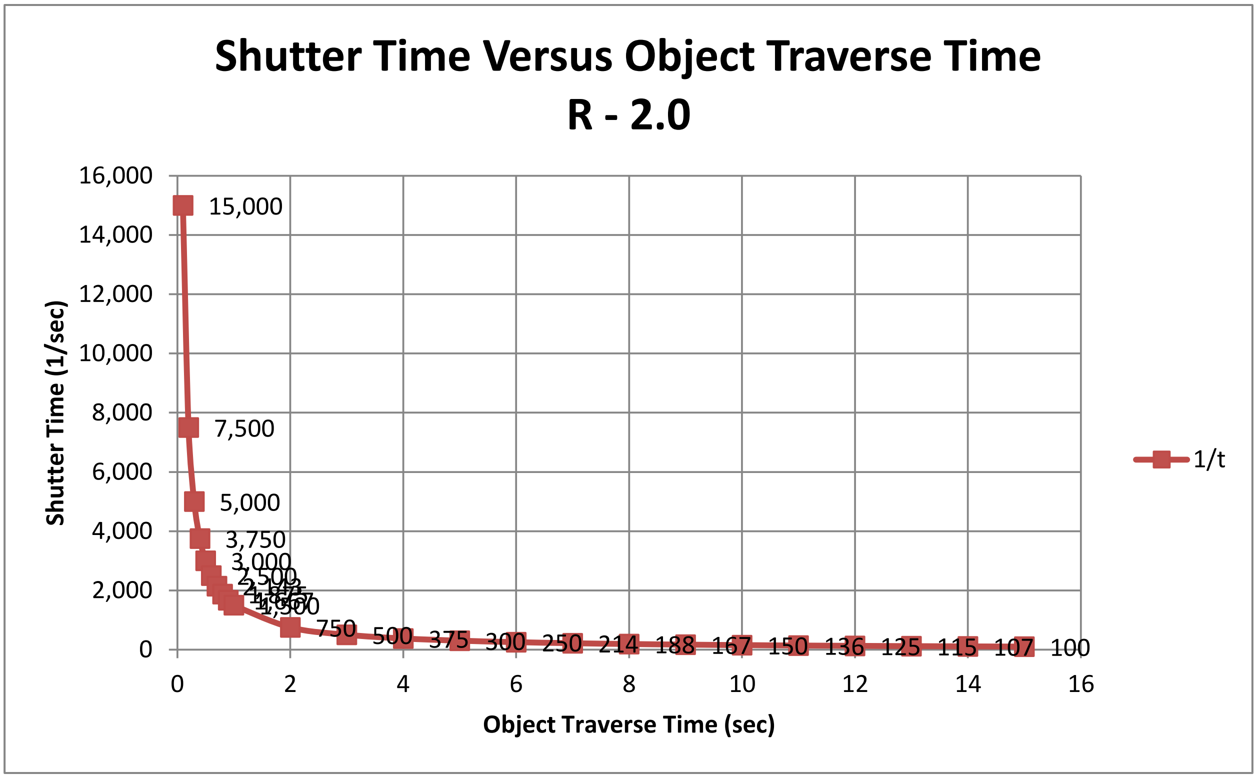

Figure 6
Shutter Time Versus Object Traverse Time

Figure 6 reveals much about "stopping motion".

1. To stop the motion of an object that requires 15 seconds to traverse the view, the minimum shutter speed would be 1/100 second.
2. To stop the motion of an object that requires 3 seconds to traverse the view, the minimum shutter speed would be 1/750 second.
3. Object Traverse Times between 3 second and 15 second result in an approximately linear variation of the shutter time.
4. Traverse times between 0.5 second and 3 seconds result in significant changes in the shutter time (1/3750 to 1/750 second).
5. Object Traverse times less than 0.5 second are not practical to measure in the field but can be roughly estimated with practice. In all cases, the shutter times are very fast and vary in a reasonably linear pattern.

Photo Dimensions			Viewing Distance	
H	W_i	Diagonal	Based Upon Diagonal; Image Viewing Ratio = 2.0	Based Upon Width; Image Viewing Ratio = 2.5
(in)	(in)	(in)	(in)	(in)
3.5	5.0	6.1	12.2	12.5
4.0	6.0	7.2	14.4	15.0
5.0	7.0	8.6	17.2	17.5
8.0	10.0	12.8	25.6	25.0
11.0	14.0	17.8	35.6	35.0
12.0	18.0	21.6	43.3	45.0
16.0	20.0	25.6	51.2	50.0
20.0	30.0	36.1	72.1	75.0

Table 5
Image Viewing Ratio Comparison

Table 5 shows a comparison of using the diagonal and the width of a photo image for computing the viewing distance. For various photographic print sizes, the viewing distance is calculated based upon,

1. The diagonal of the print and the Image Viewing Ratio equals 2.0 and
2. The width of the print and the Image Viewing Ratio equals 2.5.

The calculated viewing distances are very close. Even the distance for the 20 x 30 print is not significant considering the difference is 2.9 inches in 72.1 inches; 4% difference is not significant.